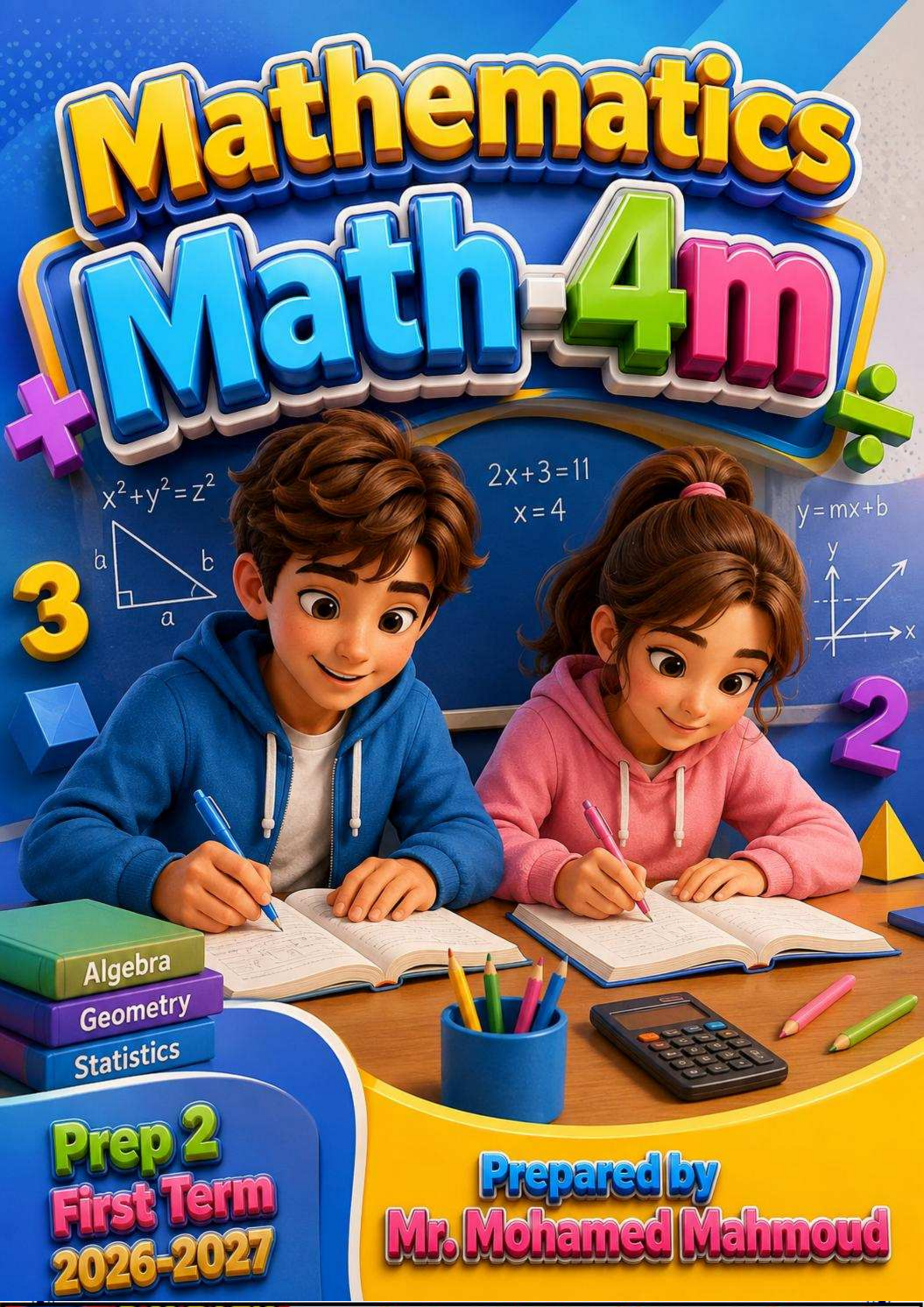
Mathematics

Math-4m



Prep 2
First Term
2026-2027

Prepared by
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Unit 1 Numbers and their Operations

First Term

Prep 2

Lesson 1

Real Numbers

Warm Up

Every day we use different kinds of numbers.

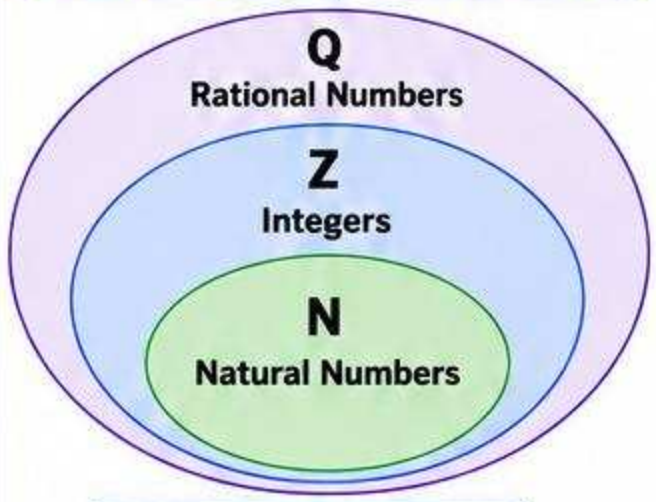
When you check your temperature, measure a length, calculate money, or estimate the distance between two cities, you are using **real numbers.**

Learning Outcomes

By the end of this lesson, you will be able to:

- Identify different sets of numbers.
- Distinguish between rational and irrational numbers.
- Classify numbers correctly.
- Understand the relationship between number sets.

The Relationship Between Sets



$$N \subset Z \subset Q$$

Remember the Number Sets

1 Natural Numbers (N)

Counting numbers beginning with zero.

$$N = \{0, 1, 2, 3, 4, 5, \ldots\}$$

Examples:

0 , 4 , 18 , 125

2 Integers (Z)

Positive numbers, negative numbers and zero.

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Examples:

-8 , -1 , 0 , 9

3 Rational Numbers (Q)

A rational number can be written in the form $\frac{a}{b}$

where $a, b \in Z$, $b \neq 0$.

Examples:

$\frac{3}{4}$, $-\frac{5}{2}$, 0.4 , 2.75 , 65% , $\sqrt[3]{36}$, $\sqrt[3]{-64}$

Irrational Numbers

An irrational number cannot be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

The set of irrational numbers is denoted by Q'.

Examples:

π , $\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt[3]{4}$

Remember

- If the square root is not a perfect square, then the result is irrational.

  Example: $\sqrt{10}$ is irrational.

- If the cube root is not a perfect cube, then the result is irrational.

  Example: $\sqrt[3]{20}$ is irrational.

Quick Check

Classify each number.

1. $\sqrt{81}$ .....
2. -7 .....
3. 0.6 .....
4. π .....

Math Fact

The decimal expansion of a rational number is either **terminating** or **recurring.**

An irrational number has a decimal expansion that never ends and never repeats.

Word Problems

1. A square field has an area of $\left(\sqrt{8}\right)^2$ square meters.

   Find the length of one side of the field.

   If a fence costs 45 pounds per meter, how much does it cost to fence the field?

2. A machine produces 2×10^5 items per day.

   The production increases 10^3 times next month.

   How many items will the machine produce per day next month?

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Key Idea

Every natural number is an integer.
Every integer is a rational number.
Therefore: $N \subset Z \subset Q$

Stage: Preparatory
Grade: Prep 2
Term: First Term

Examples of Irrational Numbers

Square root of a rational number that is not a perfect square	Cube root of a rational number that is not a perfect cube
• $\sqrt{2}$ is irrational because there is no rational number whose square equals 2. In other words, **2 is not a perfect square.**	• $\sqrt[3]{5}$ is irrational because there is no rational number whose cube equals 5. In other words, **5 is not a perfect cube.**

Other examples

$\sqrt{3}$, $\sqrt{6}$, $\sqrt{11}$, $\sqrt[3]{2}$, $\sqrt[3]{10}$, π , φ (the golden ratio), ...

Note

1 The decimal expansion of a rational number is either terminating or recurring.

Examples:

$0.25 = 0.25000...$ (terminating)

$\frac{2}{3} = 0.\overline{6}$ (recurring)

2 An irrational number has a decimal expansion that never ends and never repeats.

Examples:

$\pi = 3.1415926535...$

$\sqrt{2} = 1.414213562...$

Solved Examples

Example 1

Which of the following numbers are rational and which are irrational ? State the reason.

1 0.125 2 $\sqrt{5}$ 3 $-3\frac{1}{2}$

Solution

1 $0.125 = \frac{125}{1000} = \frac{1}{8}$ ⇒ rational (can be written as $\frac{a}{b}$)

2 $\sqrt{5}$ is irrational because 5 is not a perfect square.

3 $-3\frac{1}{2} = -\frac{7}{2}$ ⇒ rational (can be written as $\frac{a}{b}$)

Example 2

Which of the following numbers are rational and which are irrational ? State the reason.

1 $\sqrt[3]{8}$ 2 $\sqrt[3]{10}$ 3 0

Solution

1 $\sqrt[3]{8} = 2$ ⇒ rational (8 is a perfect cube)

2 $\sqrt[3]{10}$ is irrational because 10 is not a perfect cube.

3 $0 = \frac{0}{5} = 0$ ⇒ rational (for any non-zero integer b)

Example 3

Classify each number as rational or irrational.

1 $\pi + 1$ 2 $-\sqrt{18}$ 3 $\sqrt{49}$

Solution

1 $\pi + 1$ is irrational because π is irrational, and adding a rational number to an irrational number gives an irrational number.

2 $-\sqrt{18}$ is irrational because 18 is not a perfect square.

3 $\sqrt{49} = 7$ ⇒ rational

TRY it Yourself

Which of the following numbers are rational and which are irrational ? State the reason.

1 0.72 2 $\sqrt{7}$ 3 $\sqrt[3]{-27}$ 4 $-1\frac{3}{5}$ 5 $\pi - 2$
6 0 7 $\sqrt{25} + \sqrt{16}$ 8 $-\sqrt{32}$ 9 $\sqrt[3]{64}$ 10 $\sqrt{50} + \sqrt{2}$

Check Yourself 1

1 Choose the correct answer :

1 Which of the following is irrational ?
ⓐ $\sqrt{4}$ ⓑ $\sqrt{5}$
ⓒ $-\frac{2}{3}$ ⓓ 0.3

2 Which of the following is rational ?
ⓐ π ⓑ $\sqrt[3]{6}$
ⓒ $\sqrt[3]{27}$ ⓓ $\sqrt{8}$

2 Complete :

1 $\sqrt{81}$ = ...
2 $\sqrt{12}$ is ..... number.
3 $-\frac{5}{7}$ can be written as $\frac{a}{b}$ where a = ... , b = ...

3 True or False :

1 $\sqrt[3]{9}$ is an irrational number. ( )
2 Every integer is a rational number. ( )
3 $\pi = 3.1416$ (terminating decimal). ( )

4 Solve the following :

1 Write $\frac{3}{4}$ as a decimal number.
2 Write one irrational number whose square root is between 4 and 5.

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Lesson One
Real Numbers

Remember

Any irrational number lies between two rational numbers, and it can be represented on the number line by a point between these two numbers.

Key Idea

Every rational number can be written as $\frac{a}{b}$, where $a, b \in \mathbb{Z}, b \neq 0$.

The set of irrational numbers is denoted by Q'.

Finding an Approximate Value for an Irrational Number

To locate an irrational number on the number line, we find two consecutive integers between which it lies.

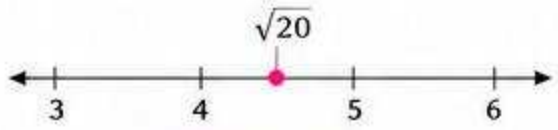
Example 2

1 $\sqrt{20}$

Since $4^2 = 16 < 20 < 25 = 5^2$

Therefore: $4 < \sqrt{20} < 5$

$\sqrt{20}$ lies between 4 and 5.



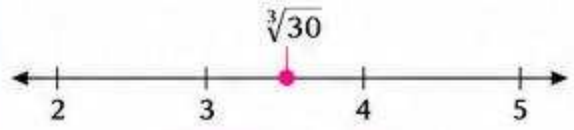
$\sqrt{20} \approx 4.47$

2 $\sqrt[3]{30}$

Since $3^3 = 27 < 30 < 64 = 4^3$

Therefore: $3 < \sqrt[3]{30} < 4$

$\sqrt[3]{30}$ lies between 3 and 4.



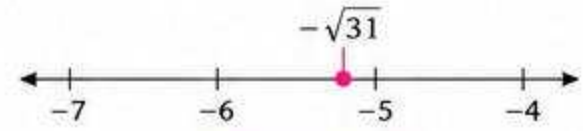
$\sqrt[3]{30} \approx 3.11$

3 $-\sqrt{31}$

Since $25 = 5^2 < 31 < 36 = 6^2$

Therefore: $5 < \sqrt{31} < 6$

$-\sqrt{31}$ lies between -6 and -5.

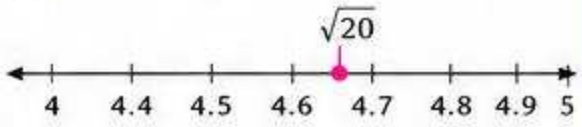


$-\sqrt{31} \approx -5.57$

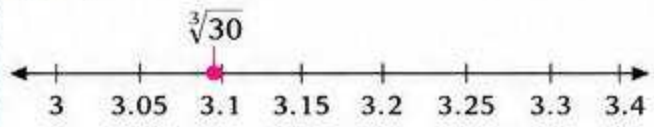
Note

The calculator can be used to find better approximate values for irrational numbers.

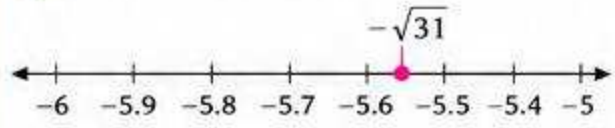
① $\sqrt{20} \approx 4.4721$



② $\sqrt[3]{30} \approx 3.1072$



③ $-\sqrt{31} \approx -5.5677$



Example 3

Estimate each of the following numbers.

1 $5\sqrt{13}$

Solution

Since $9 < 13 < 16 \Rightarrow 3 < \sqrt{13} < 4$

Multiply by 5 :

$5 \times 3 < 5\sqrt{13} < 5 \times 4$

$15 < 5\sqrt{13} < 20$

So, $5\sqrt{13} \approx 18$

2 $4 - \sqrt{50}$

Solution

Since $49 < 50 < 64 \Rightarrow 7 < \sqrt{50} < 8$

Subtract from 4 :

$4 - 8 < 4 - \sqrt{50} < 4 - 7$

$-4 < 4 - \sqrt{50} < -3$

So, $4 - \sqrt{50} \approx -3.5$

3 $\frac{2 + \sqrt{17}}{3}$

Solution

Since $16 < 17 < 25 \Rightarrow 4 < \sqrt{17} < 5$

Add 2 : $6 < 2 + \sqrt{17} < 7$

Divide by 3 : $\frac{6}{3} < \frac{2 + \sqrt{17}}{3} < \frac{7}{3}$

$2 < \frac{2 + \sqrt{17}}{3} < 2.33$

So, $\frac{2 + \sqrt{17}}{3} \approx 2.17$

Real Life

A ladder of length $\sqrt{50}$ meters leans against a wall. What is the approximate length of the ladder in meters?

Since $\sqrt{50} \approx 7.07$

The ladder length is about 7.07 m.

TRY it Yourself 1

Find two consecutive integers between which each of the following numbers lies, and determine its approximate location on the number line.

1 $\sqrt{30}$ | **2** $\sqrt[3]{18}$ | **3** $-\sqrt{27}$ | **4** $\sqrt{45}$ | **5** $\sqrt[3]{70}$ | **6** $-\sqrt{50}$

Check Yourself 2

1 Choose the correct answer :

1 Which of the following is irrational ?

ⓐ $\sqrt{26}$ ⓑ $\sqrt{35}$

ⓒ $\sqrt{46}$ ⓓ $\sqrt{50}$

2 Which of the following is rational ?

ⓐ π ⓑ $\sqrt{6}$

ⓒ $\sqrt[3]{27}$ ⓓ $\sqrt{8}$

2 Complete :

1 $\sqrt{19}$ is between ..... and .....

2 $\sqrt[3]{40}$ is between ..... and .....

3 $-\frac{5}{7}$ can be written as $\frac{a}{b}$ where a = ... , b = ...

3 True or False :

1 Every irrational number is between two integers. ( )

2 Every integer is a rational number. ( )

3 $\pi = 3.1416$ (terminating decimal). ( )

4 Solve :

1 Write $\frac{3}{4}$ as a decimal number.

2 Write one irrational number whose square root is between 4 and 5.

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Lesson One

Real Numbers

Note

To represent an irrational number on the number line, we choose two consecutive integers between which it lies, then mark the point between them according to its approximate value.

Attention

$\sqrt{a}$ is irrational if a is not a perfect square.

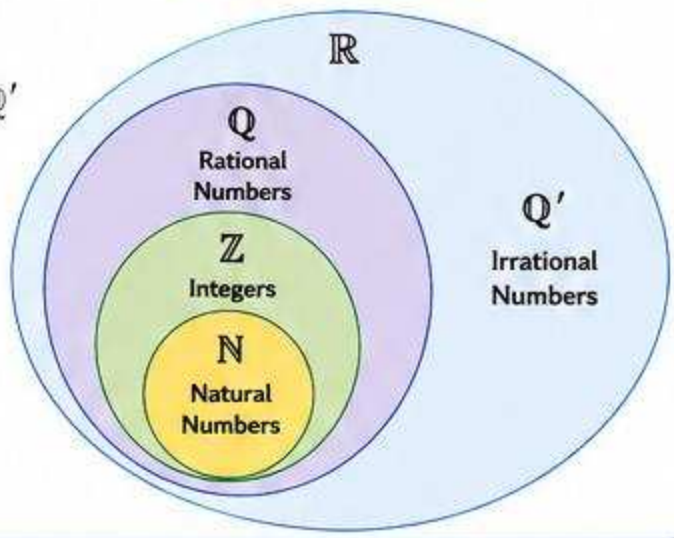
$\sqrt[3]{a}$ is irrational if a is not a perfect cube.

Example 4

Put the appropriate symbol $\in, \notin, \subset, \not\subset$ in the empty space :

1. $\sqrt{12}$ ..... $\mathbb{R}$
2. $\sqrt{16}$ ..... $\mathbb{Q}$
3. $\{\sqrt{2}, \sqrt{3}\}$ ..... $\mathbb{Q}'$
4. $\sqrt{9}$ ..... $\mathbb{Z}$
5. -3 ..... $\mathbb{Z}$
6. $\mathbb{N}$ ..... $\mathbb{Z}$
7. -5 ..... $\mathbb{Q}'$
8. π ..... $\mathbb{R}$
9. $\sqrt[3]{27}$ ..... $\mathbb{N}$
10. 0.85 ..... $\mathbb{Q}$
11. $\{0, \sqrt{8}\}$ ..... $\mathbb{N}$

Real Numbers $\mathbb{R}$



Solution

1. $\in$
2. $\in$
3. $\in$
4. $\in$
5. $\not\subset$
6. $\in$
7. $\notin$
8. $\in$
9. $\in$
10. $\in$
11. $\not\subset$

Math Fact

π (Pi) is the ratio between the circumference of a circle and its diameter and is approximately $3.1415926535\ldots$

It cannot be written as $\frac{a}{b}$.

Examples

- π (Pi)
- φ (the golden ratio) ≈ 1.6180339887
- $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \ldots$
- $\sqrt[3]{2}, \sqrt[3]{5}, \sqrt[3]{7}, \ldots$

Solving Equations

To solve equations in $\mathbb{R}$, we use the same properties of equality (addition, subtraction, multiplication, division).

Remember: Add or subtract the same number from both sides, or multiply or divide both sides by the same non-zero number.

Example 5

1 $2x^2 - 3 = 13$, where $x \in \mathbb{R}$

$2x^2 = 13 + 3 = 16$ (Add 3 to both sides)

$x^2 = 8$ (Divide by 2)

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

Solution set $= \{-2\sqrt{2}, 2\sqrt{2}\}$

2 $3x^2 + 5 = 23$, where $x \in \mathbb{R}$

$3x^2 = 23 - 5 = 18$ (Subtract 5)

$x^2 = 6$ (Divide by 3)

$x = \pm\sqrt{6}$

Solution set $= \{-\sqrt{6}, \sqrt{6}\}$

3 $\frac{1}{4}x^2 + \frac{1}{2} = 3$, where $x \in \mathbb{R}$

$\frac{1}{4}x^2 = 3 - \frac{1}{2} = \frac{5}{2}$

$x^2 = 10$ (Subtract $\frac{1}{2}$)

$x = \pm\sqrt{10}$ (Multiply by 4)

Solution set $= \{-\sqrt{10}, \sqrt{10}\}$

4 $x^2 - 4 = 0$, where $x \in \mathbb{R}$

$x^2 = 4$ (Add 4)

$x = \pm 2$

Solution set $= \{-2, 2\}$

TRY it Yourself 2

Find the solution set of each of the following equations in $\mathbb{R}$.

1. $x^2 - 5 = 20$
2. $2x^2 + 3 = 11$
3. $5x^2 + 5 = 45$
4. $\frac{1}{3}x^2 - 1 = 2$
5. $x^2 + 9 = 18$
6. $4x^2 - 4 = 0$

Check Yourself 3

1 Choose the correct answer :

1. The solution set of $x^2 = 25$ is .....
   ⓐ $\{5\}$ ⓑ $\{-5\}$ ⓒ $\{-5, 5\}$ ⓓ $\emptyset$
2. If $x^2 = 12$, then $x =$ .....
   ⓐ $2\sqrt{3}$ ⓑ $\sqrt{6}$ ⓒ $\pm 2\sqrt{3}$ ⓓ $\pm\sqrt{6}$

2 Complete :

1. $\sqrt{50}$ is between ..... and .....
2. $\sqrt[3]{20}$ is between ..... and .....
3. $\pi \approx$ ..... (to 2 decimal places)
4. $\sqrt{7} \approx$ ..... (to 2 decimal places)

3 True or False :

1. Every rational number is a real number. ( )
2. Every irrational number can be written in the form $\frac{a}{b}$. ( )
3. $\sqrt{64}$ is an irrational number. ( )
4. -9 is a natural number. ( )

4 Solve :

1. The length of a rectangular garden is $\sqrt{50}$ m. Estimate this length.
   .....
2. A triangle has perimeter $\sqrt{72}$ m. Estimate the perimeter.
   .....

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UNIT	Unit Name	Term	Lesson
1	Numbers and their Operations	First Term	Lesson One : Real Numbers

Key Idea

The decimal expansion of a rational number is either terminating or recurring.
Irrational numbers have decimal expansions that never end and never repeat.

Remember

- The set of real numbers is denoted by $\mathbb{R}$.
  $\mathbb{R} = Q \cup Q'$
  where Q is the set of rational numbers and Q' is the set of irrational numbers.

Example 6 Representing Irrational Numbers on the Number Line

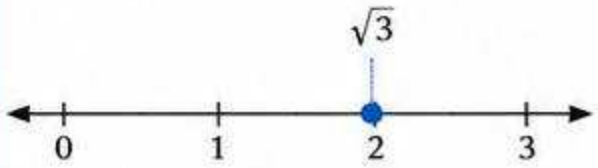
Represent each of the following numbers on the number line using the given approximate value.

1 $\sqrt{3}$, where $\sqrt{3} \approx 1.73$

Solution

$1 < 1.73 < 2$

$\sqrt{3}$ lies between 1 and 2.



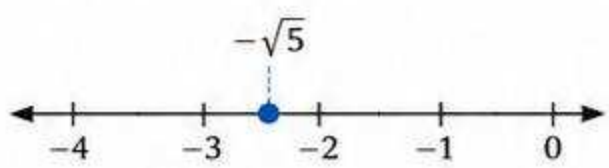
$\sqrt{3} \approx 1.73$

2 $-\sqrt{5}$, where $\sqrt{5} \approx 2.24$

Solution

$-3 < -2.24 < -2$

$-\sqrt{5}$ lies between -3 and -2.



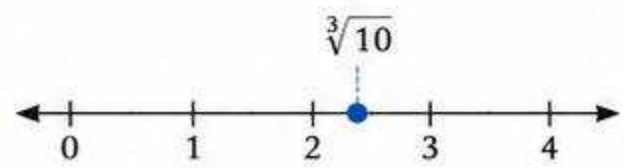
$-\sqrt{5} \approx -2.24$

3 $\sqrt[3]{10}$, where $\sqrt[3]{10} \approx 2.15$

Solution

$2 < 2.15 < 3$

$\sqrt[3]{10}$ lies between 2 and 3.



$\sqrt[3]{10} \approx 2.15$

Note

We can compare irrational numbers using their approximate values.

Since $\sqrt{2} \approx 1.41$ and $\sqrt{3} \approx 1.73$ therefore $\sqrt{2} < \sqrt{3}$

Since $\sqrt[3]{2} \approx 1.26$ and $\sqrt{2} \approx 1.41$ therefore $\sqrt[3]{2} < \sqrt{2}$

Since $\sqrt[3]{7} \approx 1.91$ and $\sqrt[3]{5} \approx 2.41$ therefore $\sqrt[3]{7} < \sqrt[3]{5}$

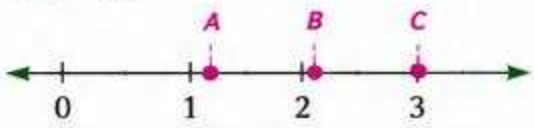
Common Irrational Numbers

- $\pi \approx 3.14159$
- $\sqrt{2} \approx 1.41421$
- $\sqrt{3} \approx 1.73205$
- $\sqrt{5} \approx 2.23607$
- $\sqrt[3]{2} \approx 1.25992$
- $\sqrt[3]{5} \approx 1.70998$

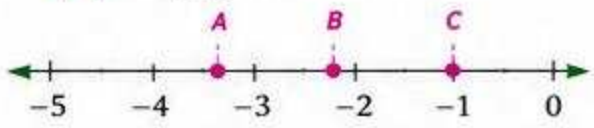
TRY it Yourself 3

Choose the correct location for each of the following numbers on the number line.

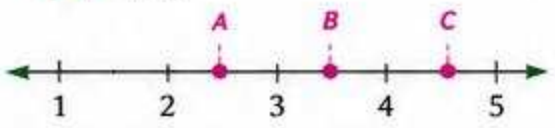
1 $\sqrt{2}$



2 $-\sqrt{10}$



3 $\sqrt[3]{20}$



Check Yourself 4

1 Choose the correct answer :

1. Which of the following is irrational ?
   ⓐ $\sqrt{36}$ ⓑ $-\frac{2}{3}$
   ⓒ $0.555\ldots$ ⓓ π
2. Which of the following is rational ?
   ⓐ $\sqrt{7}$ ⓑ $\sqrt{11}$
   ⓒ $\sqrt[3]{17}$ ⓓ $0.\overline{27}$

2 Complete :

1. $\sqrt{20}$ is between ..... and .....
2. $-\sqrt{50}$ is between ..... and .....
3. $\sqrt[3]{27}$ is between ..... and .....
4. $\sqrt{5} \approx$ ..... (to 2 decimal places)
5. $\sqrt{3} \approx$ ..... (to 2 decimal places)

3 True or False :

1. Every rational number is an integer. ( )
2. $\sqrt{8}$ is an irrational number. ( )
3. $\sqrt[3]{-27}$ is a rational number. ( )
4. -5 is a natural number. ( )

4 Solve :

1. Write an irrational number whose value is between 2 and 3.
   .....
2. The diagonal of a square floor has length $\sqrt{72}$ m. Estimate the length of the diagonal.
   .....

Word Problems

1. A rectangular hall has length $\sqrt{50}$ m and width $\sqrt{18}$ m. Estimate its perimeter.
   (Use $\sqrt{50} \approx 7.07$, $\sqrt{18} \approx 4.24$)

2. A circular garden has diameter $\sqrt{98}$ m. Estimate its circumference.
   (Use $\pi \approx 3.14$ and $\sqrt{98} \approx 9.90$)

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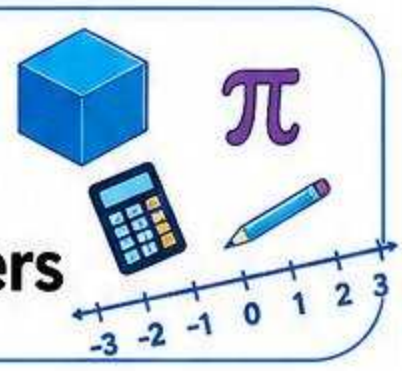
Unit 1 Numbers and their Operations

Lesson 1 Real Numbers

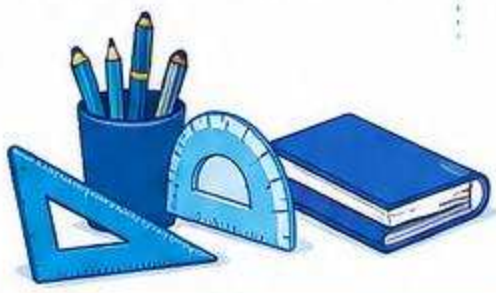


Remember | Understand | Apply | Problem Solving | From the school book

Exercise 1

Which of the following numbers are rational numbers, and which are irrational numbers ? State your reason.

1. -8
2. $\sqrt{36}$
3. $\sqrt{18}$
4. $\frac{7}{11}$
5. $-2\frac{1}{3}$
6. $\sqrt{50}$
7. 3.25
8. $\frac{3}{4}$
9. $\sqrt{2}$
10. $-\sqrt{81}$
11. $\frac{5\pi}{6}$
12. 0
13. $\sqrt[3]{-27}$
14. $2\sqrt{7}$
15. $0.606060\ldots$
16. $\frac{\sqrt{5}}{\sqrt{5}}$
17. $-\sqrt{7}$
18. $1.2\overline{3}$
19. $-\frac{9}{12}$
20. $\sqrt[3]{64}$
21. $-\sqrt{2}$
22. π
23. $\sqrt{49}-\sqrt{16}$
24. $\sqrt{3}+\sqrt{12}$



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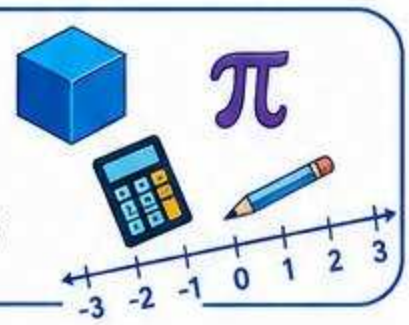
Unit 1 Numbers and their Operations

Lesson 1 Real Numbers



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Exercise 2

Put the appropriate symbol $\in$ or $\notin$ in the empty box :

1. $-7 \square \mathbb{Z}$ .....
2. $\sqrt{11} \square \mathbb{R}$ .....
3. $3.5 \square \mathbb{Q}$ .....
4. $0.7 \square \mathbb{Q}$ .....
5. $-\sqrt{9} \square \mathbb{N}$ .....
6. $\frac{5}{7} \square \mathbb{Z}$ .....

Exercise 3

Write all possible sets of the number sets ($\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$) to which each of the following numbers belongs :

1. $\sqrt{4}$ .....
2. -3 .....
3. 0.25 .....
4. $\sqrt{25}$ .....
5. $-\sqrt{16}$ .....
6. $\frac{11}{3}$ .....

Exercise 4

Find two consecutive integers between which each of the following numbers lies and determine its approximate location on the number line :

1. $\sqrt{10}$

( ) $\sqrt{10}$ ( )

2. $-\sqrt{17}$

( ) $-\sqrt{17}$ ( )

3. $\sqrt[3]{20}$

( ) $\sqrt[3]{20}$ ( )

4. $\sqrt{2}$

( ) $\sqrt{2}$ ( )

5. $-\sqrt[3]{15}$

( ) $-\sqrt[3]{15}$ ( )

6. $\sqrt{45}$

( ) $\sqrt{45}$ ( )

Exercise 5

If X is a real number , find the value of X in each of the following cases :

1. $X + 2 < \sqrt{18} < X + 3$
   $\therefore$ .....
2. $X - 1 < \sqrt{41} < X + 1$
   $\therefore$ .....
3. $X < \sqrt[3]{60} < X + 1$
   $\therefore$ .....
4. $X - 2 < \sqrt{80} < X - 1$
   $\therefore$ .....

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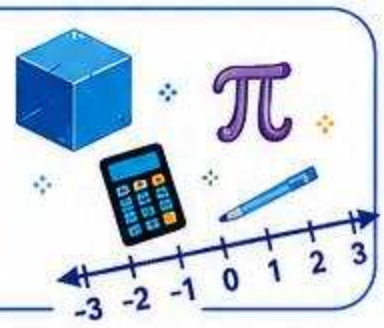
Lesson 1 Real Numbers



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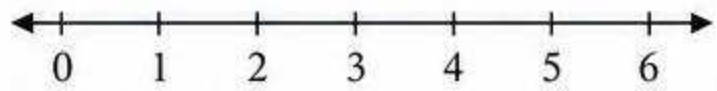
Exercise 6

Find the value of the integer n that satisfies : $n < \sqrt{20} < n + 1$, then find the approximate value of $\sqrt{20}$ to the nearest tenth , and mark its approximate location on the number line.

$n =$ _____ Approximate value of $\sqrt{20} \approx$ _____



Exercise 7

Put one of the symbols $<$, $>$ or $=$ in each box to get a true statement :

1. $\sqrt{75}$ ☐ 8.6
2. $-\sqrt{16}$ ☐ -4
3. $\sqrt{27}$ ☐ $\sqrt{9}$
4. -3 ☐ $\sqrt[3]{-27}$
5. $1-\sqrt{2}$ ☐ $\sqrt{2}-1$
6. $\sqrt[3]{8}$ ☐ 2

Exercise 8

1. Arrange the numbers : $2\sqrt{5}$, 5.1 , $\sqrt{50}$, 6 , $-\sqrt{18}$ from the smallest to the largest.

.....

2. Arrange the numbers : $\sqrt{72}$, 7 , $-\sqrt{32}$, $\sqrt{45}$ in descending order.

.....

Exercise 9

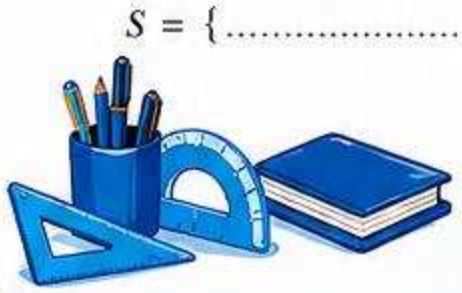
Find the solution set for each of the following equations in $\mathbb{R}$:

1. $x^3 - 8 = 0$, $S = \{$ ..... $\}$
2. $2x^3 + 5 = 13$, $S = \{$ ..... $\}$
3. $\frac{3}{4}x^2 = 27$, $S = \{$ ..... $\}$
4. $3x^2 - 2 = -5$, $S = \{$ ..... $\}$
5. $\frac{1}{3}x^2 - 4 = 0$, $S = \{$ ..... $\}$
6. $\frac{2}{x^2} + 3 = 7$, $x \neq 0$, $S = \{$ ..... $\}$
7. $(x^2 - 4)(x^2 - 9) = 0$, $S = \{$ ..... $\}$
8. $3x^3 - 1 = 26$, $S = \{$ ..... $\}$

Exercise 10

Find the solution set for each of the following equations in $\hat{\mathbb{Q}}$:

1. $4x^3 = -54$, $S = \{$ ..... $\}$
2. $2x^2 - 7 = 9$, $S = \{$ ..... $\}$
3. $5x^2 - 2 = 3x^2 + 4$, $S = \{$ ..... $\}$
4. $2(x^3 + 1) = 16$, $S = \{$ ..... $\}$
5. $(x - 2)^2 = 25$, $S = \{$ ..... $\}$
6. $(x^3 + 1)(x^2 - 4) = 0$, $S = \{$ ..... $\}$



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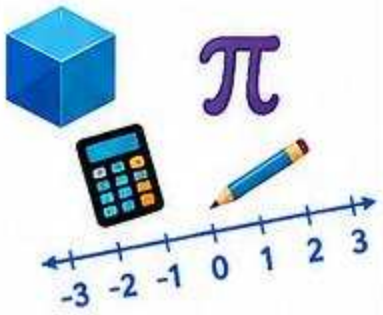
Unit 1 Numbers and their Operations

Lesson 1 Real Numbers



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Exercise 11

Find the solution set for each of the following equations :

1 $\frac{1}{2}x^2 + 3 = 0$

First : in $\mathbb{Q}$ **Second : in $\mathbb{R}$**

$S = \{$ ..... $\}$

2 $\frac{3}{4}x^2 = \frac{27}{4}$

First : in $\mathbb{Q}$ **Second : in $\mathbb{R}$**

$S = \{$ ..... $\}$

3 $2(x^2 - 1) = -8$

First : in $\mathbb{Q}$ **Second : in $\mathbb{R}$**

$S = \{$ ..... $\}$

Exercise 12

Discuss : Mona and Karim answered the following question :

Find two consecutive integers between which the irrational number $-\sqrt{7}$ lies.

Mona's answer :

$-3 < -\sqrt{7} < -2$

$\Rightarrow (-3)^2 > (-\sqrt{7})^2 > (-2)^2$

$\Rightarrow 9 > 7 > 4$

$\Rightarrow 4 < 7 < 9$

$\Rightarrow 2^2 < 7 < 3^2$

$\Rightarrow -3 < -\sqrt{7} < -2$

Karim's answer :

$-3 < -\sqrt{7} < -2$

$\Rightarrow -(-3) > -\sqrt{7} > -(-2)$

$\Rightarrow -3 > -\sqrt{7} > 2$

$\Rightarrow -(-3) < -(-\sqrt{7}) < -(-2)$

$\Rightarrow 3 < \sqrt{7} < 2$

$\Rightarrow -3 < -\sqrt{7} < -2$

Who followed the correct method ? .....

Exercise 13

What is the smallest valid integer value of the variable n that makes both numbers $\sqrt{36+n}$ and $\sqrt{2n+18}$ rational numbers ?

$n =$ .....

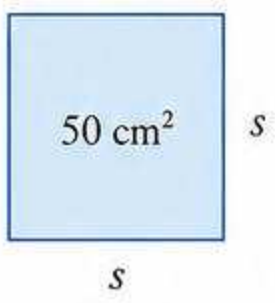
Applications

14 **Geometry :** A square has an area of 50 cm^2.

To which of the following sets of numbers does its side length belong ?

Select all possible sets.

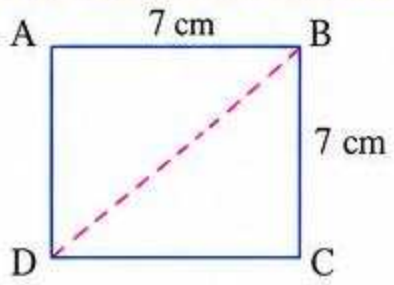
1 ☐ $\mathbb{R}$ 2 ☐ $\mathbb{Q}$ 3 ☐ $\mathbb{Q}'$ 4 ☐ $\mathbb{Z}$ 5 ☐ $\mathbb{N}$



15 **Geometry :** A square has a side length of 7 cm.

Find the length of its diagonal.

Answer : The diagonal length = ...



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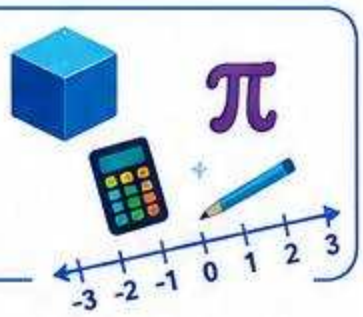
Unit 1 Numbers and their Operations

Lesson 1 Real Numbers



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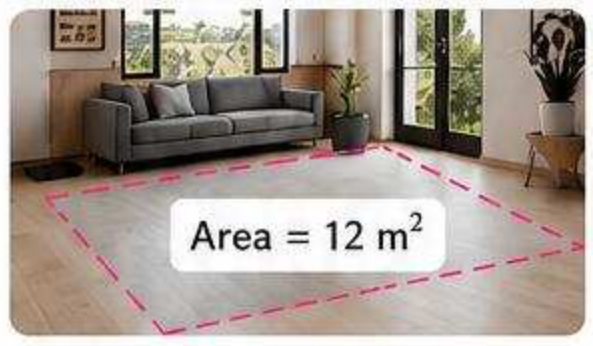
16 **Real Life :** A room has a square floor of area 12 m^2.
Ahmed wants to surround the floor with a decorative strip.
Estimate the length of this strip.

Answer : The length of the strip ≈ ..... m



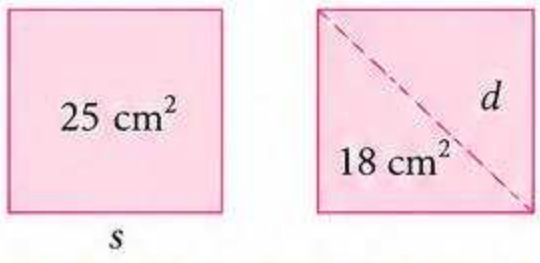
17 **Nature :** The golden rectangle is noticed in the shell of a marine organism , and the ratio of its length to its width is equal to $\frac{1+\sqrt{5}}{2}$. Estimate this value.

Answer : The value ≈ ..... (to the nearest tenth)

18 **Arts :** Samar formed a cube using clay with a total area of 294 cm^2.
Find the length of its edge.
Is the length of the edge a rational number ?

Answer : The edge length = ... cm
It is ..... number.

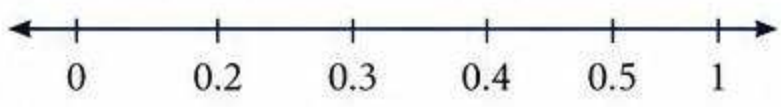
19 **Geometry :** Which is greater : The length of the side of a square with an area of 25 cm^2 or the length of the diagonal of a square with an area of 18 cm^2 ?
Explain your answer.



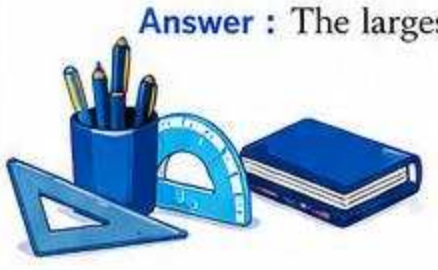
20 **Agriculture :** A rectangular garden with an area of 72 m^2, its length is twice its width.
Find the length and width to the nearest meter.

Creative Thinking

21 Find an irrational number lying between : 0.2 and 0.3 .
Answer : .....



22 If $8 < \sqrt{x} < 9$, $15 < \sqrt{y} < 16$ where x and y are positive integers.
Find the largest value of : $y - x$
Answer : The largest value of $y - x$ = ...



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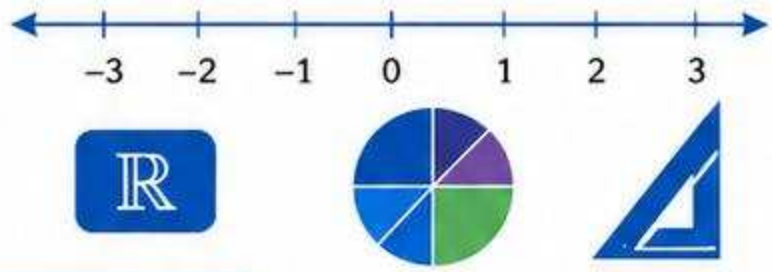
Unit 1

Lesson 1

Intervals and their Operations



Warm Up

During your previous studies, you represented subsets of natural numbers and integers using lists and set notation.

When dealing with **real numbers**, listing every element is impossible because there are infinitely many numbers between any two real numbers.

To solve this problem, mathematicians use **intervals**.

Learning Outcomes

By the end of this lesson, you should be able to:

- Define an interval.
- Distinguish between open and closed intervals.
- Write intervals using symbols.
- Represent intervals on the number line.
- Perform operations on intervals.

Recall

If $A = \{x : x \in \mathbb{R},\ -2 \leq x < 4\}$

then every real number satisfies

$$-2 \leq x < 4$$

Notice that this set cannot be written by listing all its elements because it contains infinitely many real numbers.

Therefore, we use **Interval Notation**.

Definition

An **interval** is a set of real numbers lying between two endpoints.

The endpoints may be:

- Included in the interval.
- Excluded from the interval.
- Or one endpoint may extend to infinity.

Example

Write the following set as an interval.

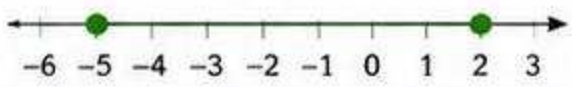
$$A = \{x : x \in \mathbb{R},\ -5 \leq x \leq 2\}$$

Solution

Both endpoints are included.

Therefore,

$A = [-5, 2]$



Basic Symbols

Symbol	Meaning	Number Line
$[\,a\,,\,b\,]$	Endpoints included	
$(\,a\,,\,b\,)$	Endpoints excluded	
$[\,a\,,\,b\,)$	Left included, right excluded	
$(\,a\,,\,b\,]$	Left excluded, right included	
$[\,a\,,+\infty\,)$	a is included, extends to $+\infty$	
$(-\infty,\,b\,]$	b is included, extends to $-\infty$	

∞ and $-\infty$ are never included in the interval.

Remember

Infinity is not a real number.

Therefore,

✓ $]-\infty, 3\,]$ is correct. ✗ $[-\infty, 3\,]$ is incorrect.

Quick Check

Write each set in interval notation.

1. $\{x : x \in \mathbb{R},\ 1 \leq x < 6\}$ _____
2. $\{x : x \in \mathbb{R},\ -4 < x \leq 3\}$ _____
3. $\{x : x \in \mathbb{R},\ x > 5\}$ _____

Think

Can an interval contain only one real number? Explain your answer.

.....

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Lesson 1 Intervals and their Operations

Types of Intervals

The following table shows the types of intervals.
When writing an interval, the smaller number must be written first.

Type of Interval	Verbal Description	Set Builder Form ($x \in \mathbb{R}$)	Interval Notation	Number Line
Closed Interval	All real numbers between a and b, including a and b.	$a \le x \le b$	$[a, b]$	
Open Interval	All real numbers between a and b, excluding a and b.	$a < x < b$	(a, b)	
Half Open Interval (left closed)	All real numbers between a and b, including a only.	$a \le x < b$	$[a, b)$	
Half Open Interval (right closed)	All real numbers between a and b, including b only.	$a < x \le b$	$(a, b]$	
Unbounded Intervals	All real numbers greater than a.	$x > a$	$(a, +\infty)$	
	All real numbers greater than or equal to a.	$x \ge a$	$[a, +\infty)$	
	All real numbers less than b.	$x < b$	$(-\infty, b)$	
	All real numbers less than or equal to b.	$x \le b$	$(-\infty, b]$	

Note

- $\mathbb{R} =]-\infty, +\infty[$
- $\mathbb{R}^+ =]0, +\infty[$
- $\mathbb{R}^- =]-\infty, 0[$
- $\mathbb{R} = \mathbb{R}^+ \cup \{0\} \cup \mathbb{R}^-$

- The set of non-negative real numbers is $\mathbb{R}^+ \cup \{0\} = [0, +\infty[$
- The set of non-positive real numbers is $\mathbb{R}^- \cup \{0\} =]-\infty, 0]$

Example 1 Write each of the following sets as an interval, then represent each on the number line.

1 $X = \{x : x \in \mathbb{R}, -3 < x \le 2\}$

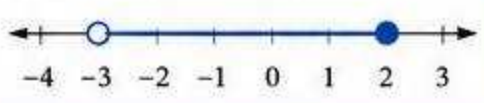
2 $Y = \{y : y \in \mathbb{R}, -2 \le y \le 1\}$

3 $A = \{a : a \in \mathbb{R}, a > 0\}$

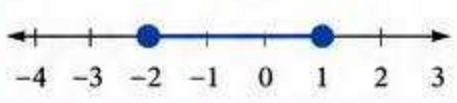
4 $B = \{b : b \in \mathbb{R}, b \le -1\}$

Solution

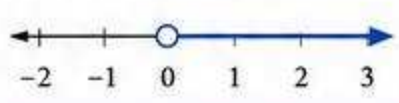
1 $(-3, 2]$



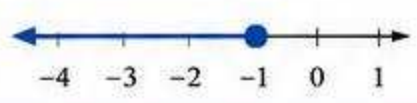
2 $[-2, 1]$



3 $(0, +\infty)$



4 $(-\infty, -1]$



TRY it Yourself 1

Write each of the following sets as an interval, then represent each on the number line.

1 $A = \{x : x \in \mathbb{R}, -2 \le x \le 4\}$

2 $Y = \{y : y \in \mathbb{R}, 0 < y \le 7\}$

3 $X = \{x : x \in \mathbb{R}, x > -5\}$

4 $C = \{c : c \in \mathbb{R}, c \le -\sqrt{9}\}$

5 The set of positive real numbers.

6 The set of non-negative real numbers.

Example 2 Choose the correct answer from those given.

1 $4 \in$ .....

(a) $]4, 7[$ (b) $[-4, 4[$ (c) $]2, 5[$ (d) $[-11, -4]$

2 $\sqrt{-8}$ ..... $[-8, -2[$

(a) $\in$ (b) $\notin$ (c) $\mathbb{C}$ (d) $\varnothing$

3 $\{\sqrt{3}, 6\}$ ..... $]1, 6[$

(a) $\in$ (b) $\notin$ (c) $\mathbb{C}$ (d) $\varnothing$

4 If $x \in [-5, \infty[$, then .....

(a) $x > -5$ (b) $x \ge -5$ (c) $x < -5$ (d) $x \le -5$

5 The sum of the real numbers in the interval $[-3, 3[$ is .....

(a) -6 (b) -3 (c) 0 (d) 6

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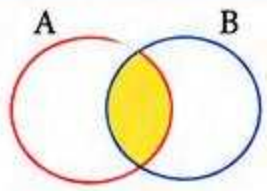
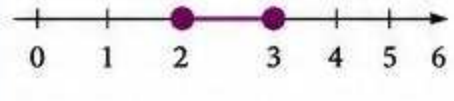
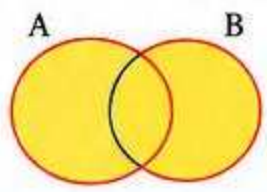
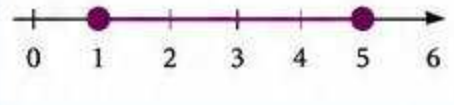
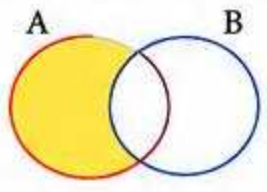
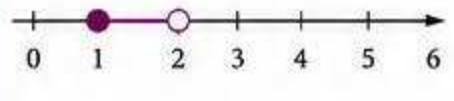
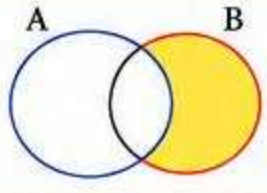
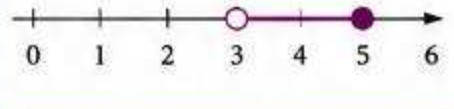
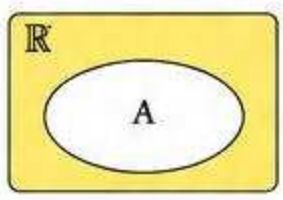
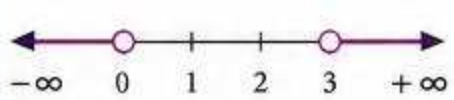
Lesson 1 Intervals and their Operations

Operations on Intervals

Intervals are subsets of real numbers, so the intersection, union, difference and complement operations can be performed on them in the same ways as on sets.

Let $A = [1, 3]$, $B = [2, 5]$

The diagrams below show how to perform operations on intervals.

Operation	Description	Venn Diagram	Number Line
1) Intersection $A \cap B$	It is the set of common elements in A and B. $A \cap B = [2, 3]$		
2) Union $A \cup B$	It is the set of elements that exist in A or in B (or in both). $A \cup B = [1, 5]$		
3) Difference $A - B$	It is the set of elements that exist in A but not in B. $A - B = [1, 2)$		
4) Difference $B - A$	It is the set of elements that exist in B but not in A. $B - A = (3, 5]$		
5) Complement $\overline{A}$	It is the set of elements that exist in $\mathbb{R}$ but not in A. $\overline{A} =]-\infty, 1[\cup]3, +\infty[$		

Note

- An interval may be empty ($\varnothing$) when there is no common element.

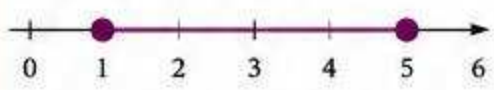
  Example :

  $[1, 2] \cap]3, 4[= \varnothing$

- The union of two intervals may be another interval or the union of two intervals disconnected.

  Examples :

  i) $[1, 2] \cup [2, 5] = [1, 5]$

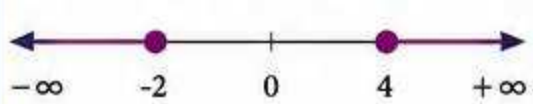
  ii) $[1, 2] \cup [4, 5] = [1, 2] \cup [4, 5]$

- To find the complement of an interval A, first find the elements outside A in $\mathbb{R}$.

  Example :

  If $A =]-2, 4[$

  Then $\overline{A} =]-\infty, -2] \cup [4, +\infty[$

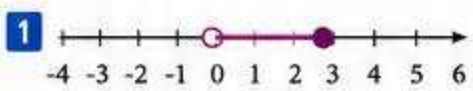
  

Example 3

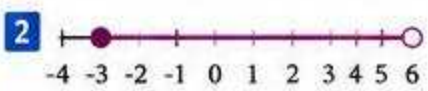
If $A = [-3, 2]$, $B =]0, 5[$ Using the number line, find all of the following :

1 $A \cap B$ 2 $A \cup B$ 3 $A - B$ 4 $B - A$ 5 $\overline{A}$ 6 $\overline{B}$

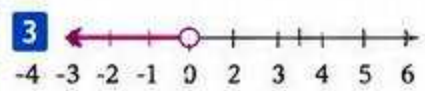
Solution

1 

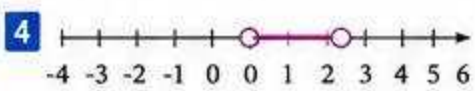
$A \cap B =]0, 2]$

2 

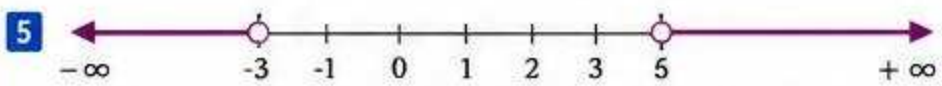
$A \cup B = [-3, 5[$

3 

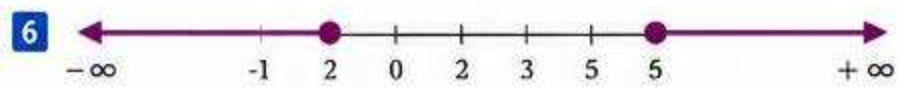
$A - B = [-3, 0]$

4 

$B - A =]2, 5[$

5 

$\overline{A} =]-\infty, -3[\cup]2, +\infty[$

6 

$\overline{B} =]-\infty, 0] \cup [5, +\infty[$

TRY it Yourself 2

If $A = [-4, 1]$, $B =]-2, 3[$ Using the number line, find all of the following :

1 $A \cap B$ 2 $A \cup B$ 3 $A - B$ 4 $B - A$ 5 $\overline{A}$ 6 $\overline{B}$

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Lesson 1 Intervals and their Operations

Example 4 Find each of the following on the number line.

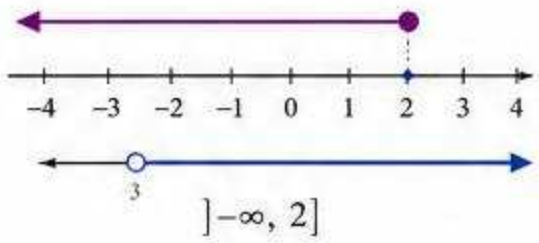
1. $]-\infty, 2] \cap]-3, \infty[$
2. $]-\infty, 3] \cup [-2, 5[$
3. $[5, \infty[-]5, \infty[$
4. $[2, \infty[\cap]-\infty, 2[$
5. $]-\infty, 4[$
6. $\mathbb{R}^+ \cap]-2, 6]$
7. $\mathbb{R} \cap]-3, 3[$
8. $\mathbb{Z} -]-\infty, 1]$
9. $\mathbb{R}^+ \cup \mathbb{R}^-$

Recall

- $\mathbb{R}^+$: set of positive real numbers $=]0, \infty[$
- $\mathbb{R}^-$: set of negative real numbers $=]-\infty, 0[$
- $\mathbb{R} = \mathbb{R}^+ \cup \{0\} \cup \mathbb{R}^-$

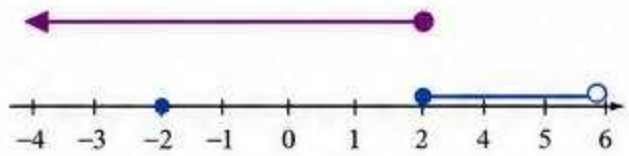
Solution

1. $]-\infty, 2]$



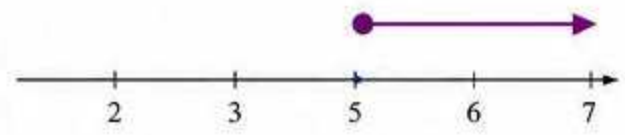
$]-\infty, 2]$

2. $]-\infty, 3] \cup [-2, 5[$



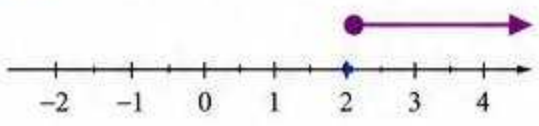
$]-\infty, 5[$

3. $[5, \infty[-]5, \infty[$



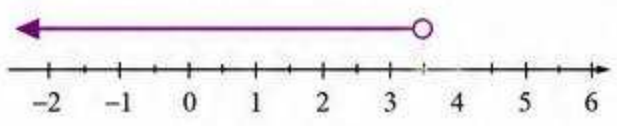
$\{5\}$

4. $[2, \infty[\cap]-\infty, 2[$



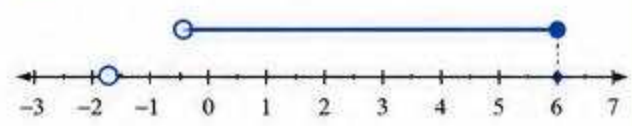
$\varnothing$

5. $]-\infty, 4[$



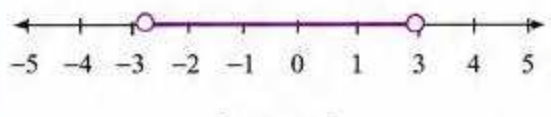
$]-\infty, 4[$

6. $\mathbb{R}^+ \cap]-2, 6]$



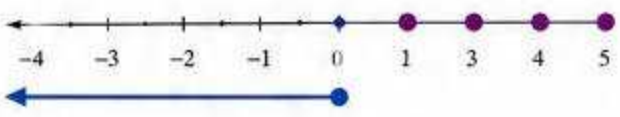
$]0, 6]$

7. $\mathbb{R} \cap]-3, 3[$



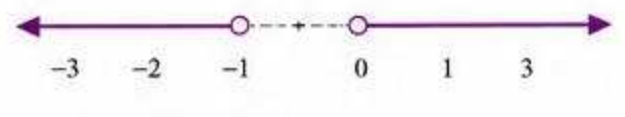
$]-3, 3[$

8. $\mathbb{Z} -]-\infty, 1]$



$\{2, 3, 4, \ldots\}$

9. $\mathbb{R}^+ \cup \mathbb{R}^-$



$]-\infty, 0[\cup]0, \infty[$

TRY it Yourself 3

If $A = [-4, 5]$, $B =]-1, 3[$ Using the number line, find all of the following :

1. $A \cap B$
2. $A \cup B$
3. $A - B$
4. $B - A$
5. $\overline{A}$
6. $\overline{B}$

Example 5 Evaluate each of the following.

1. $]-\infty, 2] \cap [-1, 3[$
2. $]-2, 4[\cup]3, 7[$
3. $[0, 2] -]0, 1[$
4. $]-\infty, -1] \cup [-1, 5]$
5. $[-3, 1[\cap [1, 6]$
6. $[-4, -1] -]-3, 0]$
7. $]-\infty, 0[\cup [0, \infty[$
8. $]-3, 2] \cap]2, 5[$
9. $\mathbb{R}^+ \cup \{0\} \cup \mathbb{R}^-$

Solution

1. $[-1, 2]$
2. $]-2, 7[$
3. $\{0, 2\}$
4. $]-\infty, 5]$
5. $\varnothing$
6. $[-4, -3]$
7. $\mathbb{R}$
8. $\varnothing$
9. $\mathbb{R}$

TRY it Yourself 4 Find all of the following.

1. $[1, 4] \cap]2, 7[$
2. $]-1, 2[\cup [2, 5]$
3. $[-3, 0] - [-1, 0[$
4. $]-\infty, 3[\cup]3, \infty[$
5. $[-2, 1[\cap]1, 4]$
6. $]-5, -1] -]-4, 0]$
7. $]-\infty, -2[\cup [-2, 6[$
8. $]-2, 2] \cap]2, 5[$
9. $\mathbb{R}^+ \cup \mathbb{R}^-$

Quick Tip

When working on the number line:

- closed dot (●) means the endpoint is included.
- open circle (○) means the endpoint is not included.

Arrows show that the interval extends to infinity.

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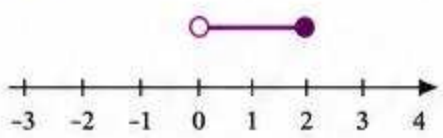
Lesson 1 Intervals and their Operations

Example 6 If $A = [-2, 1]$, $B =]0, 4[$ Using the number line, find all of the following :

1 $A \cap B$ | 2 $A \cup B$ | 3 $A - B$ | 4 $B - A$ | 5 $\overline{A}$ | 6 $\overline{B}$

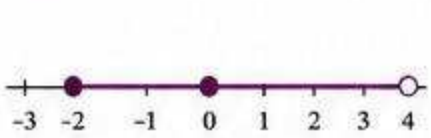
Solution

1 $A \cap B$



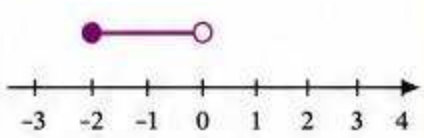
$A \cap B =]0, 1]$

2 $A \cup B$



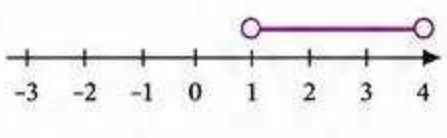
$A \cup B = [-2, 4[$

3 $A - B$



$A - B = [-2, 0[$

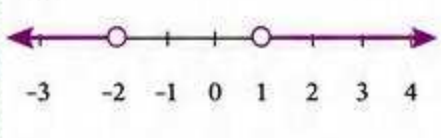
4 $B - A$



$B - A =]1, 4[$

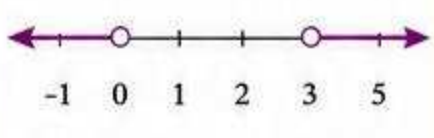
5 $\overline{A}$

$\mathbb{R}$



$\overline{A} =]-\infty, -2[\cup]1, +\infty[$

6 $\overline{B}$



$\overline{B} =]-\infty, 0] \cup [4, +\infty[$

Key Notes

- Properties of operations on intervals

Commutative Laws

$A \cap B = B \cap A$

$A \cup B = B \cup A$

Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Identity Laws

$A \cap \mathbb{R} = A$

$A \cup \varnothing = A$

Complement Laws

$A \cup \overline{A} = \mathbb{R}$

$A \cap \overline{A} = \varnothing$

TRY it Yourself 5

If $A = [-3, 2]$, $B =]1, 5[$ Using the number line, find all of the following :

1 $A \cap B$ | 2 $A \cup B$ | 3 $A - B$ | 4 $B - A$ | 5 $\overline{A}$ | 6 $\overline{B}$

Example 7 Find the value of x in each of the following :

1 If $x \in [-5, 2]$, then $-3 \leq x + 1 < 0$

2 If $x \in [1, 4]$, then $5 < x + 4 \leq 8$

3 If $x \in]-2, 3]$, then $-4 \leq 2x < 6$

4 If $x \in [-1, 5[$, then $1 < x + 2 \leq 6$

5 If $x \in [2, 7]$, then $-1 \leq 3x - 5 \leq 16$

6 If $x \in]-2, 4[$, then $-2 < x - 3 < 1$

Solution

1 $-3 \leq x+1 < 0$

Add 1 to all parts

$-3+1 \leq x-1-1 < 0+1$

$-2 \leq x < 1$

2 $5 < x+4 \leq 8$

Add 4 to all parts

$5+4 < x+4+4 \leq 8+4$

$9 < x \leq 12$

3 $-4 \leq 2x < 6$

Multiply by 2

$-4 \times 2 \leq 2x \times 2 < 6 \times 2$

$-8 \leq x < 3$

4 $1 < x+2 \leq 6$

Add 2 to all parts

$1+2 < x+2+2 \leq 6+2$

$3 < x \leq 8$

5 $-1 \leq 3x - 5 \leq 16$

Add 5 to all parts

$-1+5 \leq 3x \leq 16+5$

$4 \leq 3x \leq 21$

Divide by 3

$\frac{4}{3} \leq x \leq 7$

6 $-2 < x - 3 < 1$

Add 3 to all parts

$-2+3 < x-3+3 < 1+3$

$1 < x < 4$

TRY it Yourself 6 Find the value of x in each of the following :

1 If $x \in [-4, 1]$, then $-2 \leq x + 2 \leq 3$

2 If $x \in]2, 5[$, then $7 < x + 5 \leq 10$

3 If $x \in]-1, 6]$, then $-2 < 2x \leq 12$

4 If $x \in [0, 3]$, then $3 < x + 1 \leq 4$

5 If $x \in [-3, 2[$, then $-10 \leq 5x < 10$

6 If $x \in]-2, 1]$, then $-1 < x - 1 < 2$

Challenge

If $A = [-3, 2] \cup]4, 6[$, $B =]1, 5]$

Find :

1 $A \cap B$ | 2 $A \cup B$ | 3 $A - B$

4 $B - A$ | 5 $\overline{A}$ | 6 $\overline{B}$

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